1 Primes, Highest Common Factor and Lowest Common Multiple

Mathematical Concepts and Skills

The questions in this section allow students to acquire mathematical concepts and skills stipulated in the latest MOE syllabus for Secondary 1 Mathematics.

A wide array of questions ranging in difficulty levels can be found in this section. By working through the questions diligently, students are expected to achieve a high level of proficiency in the topic.

Question 1

Determine whether each of the following is a prime or a composite number.

(a) 31

(b) 43

(c) 51

(d) 57

(e) 73

(f) 85

Question 2

Determine whether each of the following is a prime or a composite number.

(a) 547

(b) 1307

(c) 1457

(d) 2051

(e) 2069

(f) 2163

Question 3

Determine whether each of the following statements is true or false. Explain your reasoning.

(a) If 8 is a factor of a number, then 2 and 4 are also factors of that number.

(b) If 18 is a multiple of a number, then 9 is also a multiple of that number.

Question 4

If a and b are whole numbers such that $a \times b = 29$, find the value of $a + b$. Explain your answer.

Question 5

(a) Write down all the prime numbers less than 15.

(b) Hence, write down two possible primes in part (a) such that their sum is also a prime.

Question 6

If $6174 = 2^a \times 3^b \times 7^c$, find the values of a, b and c.

Question 7

(a) Express 225 as a product of its prime factors.

(b) Hence or otherwise, find the values of x and y for which $225 = 3^x 5^y$.

Question 8

Find the square root of $28 \times 35 \times 45$ using prime factorisation.

Question 9

(a) Express 576 as a product of prime factors, giving your answer in index notation.

(b) Hence, find the square root of 576.

Question 10

(a) Using prime factorisation, express 784 as a product of prime factors using index notation.

(b) Hence, find the value of $\sqrt{784}$.

Question 11

(a) Express 34969 as a product of its prime factors.

(b) Hence, express $\sqrt{34969\times 100}$ as a product of its prime factors.

Question 12

(a) Express 4900 as a product of its prime factors.

(b) Hence, evaluate $\dfrac{\sqrt{4900}}{42}$.

Question 13

(a) Express 4410 as a product of prime factors in index notation.

(b) Hence, evaluate $\sqrt{44100}$.

Question 14

(a) Express 156 as a product of its prime factors, giving your answers in index notation.

(b) Hence, without using a calculator, find the value of $\sqrt{156\times 39}$.

Question 15

Given that $(6\times 7)^3 = 74088$, find $\sqrt[3]{74088}$.

Question 16

(a) Express 9261 as the product of prime factors in index notation.

(b) Use the result in part (a) to find the cube root of 9261.

Question 17

(a) Express 5832 as a product of prime factors.

(b) Hence, find the value of $\sqrt[3]{5832}$.

Question 18

(a) Express 2744 as a product of its prime factors. Leave your answer in index notation.

(b) Hence, evaluate $\sqrt[3]{2744}$. Show your working clearly.

Question 19

(a) Express 3375 as a product of prime factors, giving your answer in index notation.

(b) Using your answer in part (a), find $\sqrt[3]{-3375}$.

Question 20

Showing all your steps clearly,

(a) find the value of c if $4536 = 2^a \times 3^b \times 7^c$

(b) find the sum of the first four prime numbers that end with the digit 7

Question 21

(a) Find the prime factorisation of 1225.

(b) Hence, find the value of

(i) $\sqrt{1225}$

(ii) $\sqrt[3]{\dfrac{1225\times 8}{25\div 7}}$

Question 22

Given that:

$$225 = 3^2 \times 5^2$$
$$5832000 = 2^6 \times 3^6 \times 5^3$$

Using the above prime factors and showing all workings clearly, find the value of

$$\sqrt[3]{-5832000} \div \sqrt{225}$$

Leave your answer in index form.

Question 23

Estimate the value of each of the following.

(a) $\sqrt{51}$

(b) $\sqrt{120}$

(c) $\sqrt{166}$

(d) $\sqrt[3]{28}$

(e) $\sqrt[3]{129}$

(f) $\sqrt[3]{8050}$

Question 24

(a) Express 1960 as a product of prime factors in index notation.

(b) Find the smallest positive integer value of m such that $1960m$ is a perfect square.

Question 25

(a) Express 3150 as a product of its prime factors, giving your answer in index notation.

(b) Find the least integer value of n given that $3150n$ is a perfect square.

Question 26

(a) Express 5040 as a product of its prime factors, giving your answer in index notation.

(b) Hence or otherwise, find the smallest integer value of x for which $5040x$ is a perfect square.

Question 27

(a) Find the square of $2^3 \times 3^6 \times 5^2$, giving your answer in index notation.

(b) Is it a perfect cube? Why?

Question 28

(a) Find the prime factorisation of 2700 and express them in index notation.

(b) Find the smallest number that must be multiplied to 2700 to make it a perfect cube.

Question 29

By using prime factorisation, find the smallest value of m when $882m$ is a cube.

Question 30

Find the smallest integer value of k such that $6534k$ is a perfect cube.

Question 31

(a) Find the prime factorisation of 1728, giving your answer in index notation.

(b) Hence, calculate the value of $\sqrt[3]{1728}$.

(c) Given that $\dfrac{1728}{n}$ is a perfect square, find the smallest value of n.

Question 32

(a) Express 336 as a product of its prime factors, giving your answer in index notation.

(b) Find the smallest integer value of p such that $336p$ is a multiple of 180.

Question 33

(a) Express 60 and 825 as products of their prime factors, giving your answer in index notation.

(b) Find the smallest positive integer value of n for which $60n$ is a multiple of 825.

Question 34

(a) Express 1521 as a product of prime factors, giving your answer in index notation.

(b) Hence, find $\sqrt{1521}$.

(c) Write down the smallest integer value of k, such that $1521k$ is a multiple of 6.

Question 35

The number 360 is written as the product of its prime factors.

$$360 = 2^3 \times 3^2 \times 5$$

(a) Write 108 as the product of its prime factors.

(b) Hence, find the smallest positive value of x for which $108x$ is a multiple of 360.

(c) Find the smallest positive integer value of y such that $360y$ is a perfect cube.

Question 36

(a) Find the second largest factor of 63.

(b) Find the 100th common multiple of 3 and 4.

Question 37

Find the smallest number that is divisible by all prime numbers between 1 and 14.

Question 38

(a) Express each of the numbers 84 and 126 as a product of their prime factors.

(b) Find

(i) the highest common factor of 84 and 126

(ii) the lowest common multiple of 84 and 126

Question 39

Find the HCF and LCM of 102 and 138.

Question 40

Find the HCF and LCM of 112 and 280.

Question 41

Find the HCF and LCM of 14, 27 and 63.

Question 42

Find the HCF and LCM of the two numbers $2^4 \times 3^5 \times 7^2 \times 13$ and $3^3 \times 5^2 \times 13^2$, giving your answer in index notation.

Question 43

Find the HCF and LCM of 1260 and $2^3 \times 3 \times 7^2 \times 11^2$. Express your answers in index notation.

Question 44

The HCF and LCM of three numbers are 18 and $2^2 \times 3^3 \times 7 \times 11$ respectively. Given that two of the numbers are 54 and 126, find the third number.

Question 45

The numbers 168 and 324, written as the products of their prime factors, are:

$$168 = 2^3 \times 3 \times 7$$
$$324 = 2^2 \times 3^4$$

Find the largest integer which is a factor of both 168 and 324.

Question 46

Find the HCF and LCM of the two numbers:

$$2^3 \times 3^3 \times 7$$
$$2^4 \times 3^2 \times 5$$

Give your answer in index notation.

Question 47

Find the HCF and LCM of the two numbers:

$$2^3 \times 3 \times 5^2 \times 13 \times 17 \times 19^2$$
$$2^2 \times 3^3 \times 7^3 \times 13^2 \times 19^3$$

Give your answer in index notation.

Question 48

(a) Find the HCF of 90, 126 and 198.

(b) Find the LCM of $2^3 \times 3^2 \times 5 \times 7$, $2^2 \times 3^3 \times 5^2 \times 11$ and $2^2 \times 3 \times 5 \times 11$, leaving your answer in index notation.

Question 49

The numbers 1575 and 42, written as the product of their prime factors, are:

$$1575 = 3^2 \times 5^2 \times 7$$
$$42 = 2 \times 3 \times 7$$

Find

(a) the largest integer which is a factor of both 1575 and 42

(b) the smallest positive integer value of n for which $42n$ is a multiple of 1575.

Question 50

(a) Express 126 as the product of its prime factors, giving your answer in index form.

(b) Find the smallest positive integer value of n for which $126n$ is a multiple of 35.

(c) The lowest common multiple of 6, 14 and x is 126. Find the two possible values of x which are odd numbers.

Question 51

Given that:

$$A = 2^2 \times 3 \times 7^4$$
$$B = 2^2 \times 3^3 \times 5^2$$

Find, in index notation,

(a) the HCF of A and B

(b) the LCM of A and B

(c) the square root of the product of A and B

Question 52

Given that

$$A = 2^2 \times 3^4 \times 5^2 \times 11^3$$
$$B = 2^3 \times 3^5 \times 11^2$$
$$C = 3^3 \times 5^4 \times 7^2 \times 11$$

Find, in index notation,

(a) the HCF of A and B

(b) the LCM of A and C

Question 53

(a) Express the following two numbers as a product of prime factors. Write your answer in index notation.

(i) 378

(ii) 1008

(b) Hence, write down the largest integer which is a factor of 378 and 1008.

Question 54

Given the following three terms:

$$x^2 \times y \times z$$
$$w \times x \times y \times z$$
$$w^2 \times x^3 \times y^2 \times z$$

(a) Find the highest common factor.

(b) Find the lowest common multiple.

(c) The sum of the HCF and LCM is 24060. Given that $w = 2$, $x = 5$ and $y = 4$, find the value of z.

Question 55

(a) Find the HCF and LCM of the following numbers: 63, 105, 420. Leave your answers in index notation form.

(b) Hence or otherwise, evaluate the HCF and LCM of the following:

$$63a^2bc,\ 105ac^3,\ 420a^5b^3c^2$$

Question 56

When the LCM of the numbers 135, 120 and 190 is divided by the HCF, the result is x. Find x.

Question 57

(a) Express 90 as a product of its prime factors.

(b) The lowest common multiple of 6, 15 and x is 90. Given that x is an odd number, find the two possible values of x.

Question 58

(a) Express 504 and 972 as products of their prime factors.

(b) Find the largest integer which is a factor of both 504 and 972.

(c) Find the smallest positive integer value of m for which $\sqrt[3]{972m}$ is a whole number.

Question 59

(a) Given that $1500x$ is a perfect square, write down the smallest possible value of x.

(b) Find the lowest common multiple of 1500 and $2\times3^2\times5^2\times7$, leaving your answer in index form.

Question 60

The numbers 180 and 784, written as the products of their prime factors, are:

$$180=2^2\times3^2\times5$$
$$784=2^4\times7^2$$

Find

(a) the highest common factor of 180 and 784

(b) the smallest positive integer value of m for which $\sqrt[3]{180\times784\times m}$ is a whole number

(c) the smallest positive integer value of n for which $180n$ is a multiple of 16

Question 61

(a) Express 1400 as a product of its prime factors. Give your answer in index notation.

(b) Hence, find the smallest positive integer value of m such that $1400m$ is a perfect square.

(c) The number 560, written as the product of its prime factors, is $560=2^4\times5\times7$. Find the highest common factor of 560 and 1400.

Question 62

(a) Showing your method clearly, express 180 as a product of prime factors. Give your answer in index notation.

(b) Using your answer in part (a), find the smallest value of k such that $180k$ is a cube.

(c) Given that $576=2^6\times3^2$, find the largest integer which is a factor of both 180 and 576.

Question 63

Expressed as the product of prime factors,

$$162=2\times3^4 \text{ and } 98=2\times7^2$$

Use these results to find

(a) the lowest common multiple of 162 and 98

(b) $\sqrt{15876}$

Leave both answers as products of their prime factors.

Question 64

The numbers 504 and 972, written as the product of their prime factors, are:

$$504=2^3\times3^2\times7$$
$$972=2^2\times3^5$$

Find the

(a) largest integer which is a factor of both 504 and 972

(b) smallest positive integer value of n for which $504n$ is a multiple of 972

Question 65

The numbers 168 and 378, written as the products of their prime factors, are:

$$168 = 2^3 \times 3 \times 7$$
$$378 = 2 \times 3^3 \times 7$$

Find

(a) the largest integer which is a factor of both 168 and 378

(b) the smallest positive integer value of n for which $168n$ is a multiple of 378

(c) the value of $\sqrt{168 \times 378}$

Problems in Real-World Contexts

The questions in this section allow students to apply the mathematical concepts and skills acquired in the previous section to problems in real-world contexts.

As stipulated in the syllabus, students are expected to be familiar with solving mathematical problems in everyday life, including travel plans, transport schedule, sports and games, recipes, floor plans, personal and household finance etc.

Question 66

Three different bells were set to chime every 4, 16 and 18 minutes respectively. Given that all three bells chimed together at 06 45 this morning, when is the next time all three bells will chime together again? Give your answer in the 24-hour notation.

Question 67

Three bells toll at intervals of 12 seconds, 30 seconds and 66 seconds respectively.

(a) If they toll together at 6 a.m., at what time will they next toll together again?

(b) How many times, including the first time at 6 a.m., will all three bells toll together between 6 a.m. and 7 a.m.?

Question 68

The traffic lights timings at 3 junctions A, B and C are programmed such that the lights turn green at 45 s, 60 s and 120 s intervals. If the lights turn green at the same time at 10 00, at what time will the lights next turn green together?

Question 69

Three traffic lights along a street turn red at regular intervals of 1 minute, 1 minute 10 seconds and 1 minute 18 seconds respectively. If all the traffic lights turn red at the same time at 08 30, when is the next time this occurs again?

Question 70

Three lighthouses flash their lights every 20 seconds, 30 seconds and 45 seconds respectively.

(a) Given that they flash together at 7 p.m., when will they next flash together?

(b) How many times will they flash together in 6 hours?

Question 71

An advertisement signboard has 3 different coloured neon lights that flash at different intervals. The blue neon light flashes every 20 seconds, the orange neon light flashes every 36 seconds and the green neon light flashes every 55 seconds. If all three neon lights flash together at 15 45, find the next time that they will flash together again.

Question 72

Three bus services operate from the same depot. The first service leaves at 8 minutes intervals, the second at 14 minutes intervals and the third at 16 minutes intervals. All three services leave the depot together at 06 00. Find the time when the three services next leave the depot together.

Question 73

(a) Find the lowest common multiple of 12, 16 and 20.

(b) Buses on each of three separate routes start at 05 55. On one route, buses leave every 12 minutes. On the second route, buses leave every 16 minutes. On the third route, buses leave every 20 minutes. At what time will the buses on each of the three routes leave together again?

Question 74

Bus A, Bus B and Bus C leave the interchange every 35, 40 and 42 minutes respectively. Given that the 3 buses leave the interchange together at 06 20, when is the next time that the 3 buses will leave the interchange together?

Question 75

Four buses operate from the same depot. The first service leaves at 3 minutes intervals, the second service at 6 minutes intervals, the third at 16 minutes intervals and the fourth at 20 minutes intervals. All four services leave the depot together at 07 30. Find the time when the four services next leave the depot again.

Question 76

The Rail Hotel provides 3 free shuttle bus services to Orchard Road, the Zoo and the Esplanade. Bus A leaves the hotel at 12 minutes intervals. Bus B leaves at 30 minutes intervals. Bus C leaves at 1 hour intervals. The three bus services start at 09 30. Find the time when the buses for the three services next leave the hotel at the same time.

Question 77

A hospital provides 3 free shuttle services to 3 different MRT stations. Shuttle A leaves the hospital at 14 minutes intervals, Shuttle B leaves at 21 minutes intervals and Shuttle C leaves at 35 minutes intervals. On a particular day, all the three services left the hospital at 10 35.

(a) How much time, in minutes, would have passed when the three services next leave the hospital at the same time again?

(b) At this point in time, Keith was looking at his watch. What was the time shown on his watch?

Question 78

(a) Find the LCM of 8, 14 and 28.

(b) 3 sprinklers are programmed such that they are switched on every 8 minutes, 14 minutes and 28 minutes respectively. How many more times, not counting the first, will the 3 sprinklers go off simultaneously if they are left on for 4.5 hours?

Question 79

Alvin and Henry are running around Jurong Park. They take 72 seconds and 90 seconds to complete a round respectively. They begin at the starting point at the same time.

(a) How long does it take for them to meet at the starting point again?

(b) How many laps will each boy have run by then?

Question 80

Three racing cars go around a track in 48, 60 and 78 seconds respectively. If they start from the same point, how many minutes would have passed before they are side by side again?

Question 81

The dimensions of a rectangular box are 168 cm by 132 cm by 84 cm. The box is to be filled with identical cubes so that there will be no empty space.

(a) Find the longest possible length of each side of a cube and hence its volume.

(b) Deduce the number of cubes that the box can contain.

Question 82

Mrs. Ho buys a piece of bean curd in the form of a rectangular block of length 112 mm, breadth 98 mm and height 84 mm from the market. She wants to divide it into equal cubes with the length of each side be $L\text{ mm}$. What is the largest possible value of L?

Question 83

Freddy has 126 cm of red wire, 140 cm of blue wire and 154 cm of green wire. He has to cut the three different coloured wires into pieces such that all of them have the same length. Deduce the greatest possible length for the pieces if there are no wires left after cutting.

Solution 84

Dan has to design a box in the shape of a cube to store rectangular bricks of dimensions 45 cm by 21 cm by 15 cm. The length of each side of the cube is d cm . To save cost, he must ensure that the bricks fit exactly into the box, leaving no gaps in between. What is the smallest possible value of d ?

Question 85

Lamp posts are positioned at intervals of 120 m along a road and pots of bougainvillea plants are placed at intervals of 96 m. The first pot of bougainvillea plant is placed at the foot of the first lamp post.

(a) If the last pot of bougainvillea plant is placed at the foot of a lamp post, find the shortest possible distance between the first and the last pot of plant.

(b) Find the total number of lamp posts and pots of bougainvillea plants within this road segment.

Question 86

Benches, lamp posts and hydrants are found at 15 m , 20 m and 30 m intervals respectively along a straight road. At the beginning of the straight road, all three objects are in line.

(a) At what distance from the beginning of the straight road will the three objects next appear in line?

(b) If the straight road is half a kilometre long, how many times do the objects appear in line?

Question 87

Lamp posts, dustbins and benches are placed at intervals of 70 m , 56 m and 84 m respectively along a stretch of road 5 km long. At the start of the road, a lamp post, a dustbin and a bench are placed together. When all three objects are placed together, they are painted red.

(a) How far away from the initial position will the next 3 red objects be spotted again?

(b) How many lamp posts are painted red?

(c) Deduce the number of lamp posts, dustbins and benches along the road.

Question 88

The inter-class volleyball games for the Secondary 1, 2 and 3 classes are held at an interval of 14 days, 21 days and 28 days respectively.

(a) On the first day, all 3 levels held their games simultaneously. Find the number of days it takes for the inter-class games for the 3 levels to be next held simultaneously.

(b) Find the maximum number of times in a year when the inter-class games for the 3 levels can be held simultaneously.

Question 89

Bernard bought 3 kinds of lollipops at the supermarket. There were 120 chocolate-flavoured lollipops, 109 orange-flavoured lollipops and 85 cola-flavoured lollipops. He ate one orange-flavoured lollipop and one cola-flavoured lollipop and packed the remaining lollipops into smaller packets. The packets were identical, each containing the same number of each type of lollipop. Find the largest possible number of packets Bernard could have packed and hence, the number of cola-flavoured lollipops there would be in each packet.

Question 90

18 red pens, 54 blue pens and 36 black pens were evenly distributed to some students. Find the largest possible number of students who received the pens.

Question 91

Kathy distributed the same number of sweets and the same number of biscuits to each of her classmates at her birthday party. She gave out 220 sweets and 300 biscuits in total. Find the largest possible number of classmates at the party.

Question 92

Gary distributed 98 strawberry sweets, 42 orange sweets and 140 lemon sweets equally to a group of students.

(a) Calculate the largest possible number of students in the group.

(b) Hence, calculate the total number of sweets each student received.

Question 93

33 girls and 58 boys volunteered to do beach-cleaning. On the day of the beach-cleaning, 1 girl and 2 boys were absent. The remaining girls and boys were divided into as many groups as possible. Each group had the same number of girls and the same number of boys. What was the largest number of groups that could be formed?

Question 94

To play a treasure hunt game, 60 girls and 105 boys were divided into groups with the same ratio of girls to boys in each group.

(a) Find the largest number of groups that could be formed.

(b) How many girls and boys were there in each group?

Question 95

On International Friendship Day, 24 students from Malaysia, 18 students from Indonesia and 12 students from Korea were involved. They were divided into groups with equal distribution of students from each country to oversee the stations.

(a) What was the greatest number of groups that could be formed?

(b) How many students from Korea would there be in each group?

Question 96

A group of 145 girl guides and 306 boy scouts were selected to perform at the National Day Parade. Two days prior to the performance, one of the girl guides sprained her ankle and had to withdraw from the performance. The remaining girl guides and boy scouts were to be divided into groups such that each group contained x girl guides and y boy scouts.

(a) What was the largest number of groups that could be formed?

(b) Hence, state the values of x and y.

Question 97

Samuel is 3 years older than his brother Chris. The HCF of their ages is 3 and the LCM of their ages is 168. How old are Samuel and Chris?

Question 98

A security guard in Block A starts his next patrol 30 minutes after he has finished each patrol. He takes 18 minutes to finish each patrol. Another security guard in Block B starts his patrol every 40 minutes, regardless of the duration of his previous patrol. Suppose the two security guards have just started their patrol at 7 p.m., find the time that they will next start their patrol at the same time again.

Question 99

A pastry chef bakes some egg tarts. If he packs the egg tarts into boxes of 3, 5 or 11, there will always be one box which is short of one egg tart. Find the smallest possible number of egg tarts baked.

Question 100

There are some magazines in a store. If they are arranged in stacks of 18, 25 or 30, there will always be a remainder of 7 magazines. What is the smallest possible number of magazines in the store?

Question 101

Mr. Yeo is the manager of a shopping mall. He has to prepare the prizes for a lucky draw. The prizes consist of vouchers of different denominations shown in the table below.

Voucher denomination	Number of vouchers available
$2	300
$5	200
$10	160

Mr. Yeo has to ensure that each prize consists of the same number of vouchers of the same denomination. Find

(a) the greatest number of prizes that can be given out

(b) the number of vouchers in a prize

(c) the total value of the vouchers in a prize

1 Primes, Highest Common Factor and Lowest Common Multiple

Mathematical Concepts and Skills

Solution 1

(a) 31 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 31 is $3+1=4$ which is not divisible by 3, then 31 is not divisible by 3.
The last digit of 31 is neither 0 nor 5, so 31 is not divisible by 5.

Prime numbers less than 31:
2, 3, 5, 7, 11, 13, …

Use a calculator to test whether 31 is divisible by prime numbers less than 31.

Since 31 is not divisible by any prime numbers less than 31, then 31 is a prime number.

(b) 43 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 43 is $4+3=7$ which is not divisible by 3, then 43 is not divisible by 3.
The last digit of 43 is neither 0 nor 5, so 43 is not divisible by 5.

Prime numbers less than 43:
2, 3, 5, 7, 11, 13, 17, 19, …

Use a calculator to test whether 43 is divisible by prime numbers less than 43.

Since 43 is not divisible by any prime numbers less than 43, then 43 is a prime number.

(c) 51 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 51 is $5+1=6$ which is divisible by 3, therefore 51 is divisible by 3 (divisibility test for 3).
Hence, 51 is a composite number.

(d) 57 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 57 is $5+7=12$ which is divisible by 3, therefore 57 is divisible by 3 (divisibility test for 3).
Hence, 57 is a composite number.

(e) 73 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 73 is $7+3=10$ which is not divisible by 3, then 73 is not divisible by 3.
The last digit of 73 is neither 0 nor 5, so 73 is not divisible by 5.

Prime numbers less than 73:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, …

Use a calculator to test whether 73 is divisible by prime numbers less than 73.

Since 73 is not divisible by any prime numbers less than 73, then 73 is a prime number.

(f) 85 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 85 is $8+5=13$ which is not divisible by 3, then 85 is not divisible by 3.
The last digit of 85 is 5, so 85 is divisible by 5.
Hence, 85 is a composite number.

Solution 2

(a) 547 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 547 is $5+4+7=16$ which is not divisible by 3, then 547 is not divisible by 3.
The last digit of 547 is neither 0 nor 5, so 547 is not divisible by 5.

$\sqrt{547}=23.4$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{547}$ is 23.

Prime numbers less than or equal to 23:
2, 3, 5, 7, 11, 13, 17, 19, 23

Use a calculator to test whether 547 is divisible by prime numbers less than or equal to 23.

Since 547 is not divisible by any of the prime numbers less than or equal to 23, then 547 is a prime number.

(b) 1307 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 1307 is $1+3+0+7=11$ which is not divisible by 3, then 1307 is not divisible by 3.
The last digit of 1307 is neither 0 nor 5, so 1307 is not divisible by 5.

$\sqrt{1307}=36.2$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{1307}$ is 31.

Prime numbers less than or equal to 31:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31

Use a calculator to test whether 1307 is divisible by prime numbers less than or equal to 31.

Since 1307 is not divisible by any of the prime numbers less than or equal to 31, then 1307 is a prime number.

(c) 1457 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 1457 is $1+4+5+7=17$ which is not divisible by 3, then 1457 is not divisible by 3.
The last digit of 1457 is neither 0 nor 5, so 1457 is not divisible by 5.

$\sqrt{1457}=38.2$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{1457}$ is 37.

Prime numbers less than or equal to 37:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37

Use a calculator to test whether 1457 is divisible by prime numbers less than or equal to 37.

1457 is divisible by 31. Hence, 1457 is a composite number.

(d) 2051 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 2051 is $2+0+5+1=8$ which is not divisible by 3, then 2051 is not divisible by 3.
The last digit of 2051 is neither 0 nor 5, so 2051 is not divisible by 5.

$\sqrt{2051}=45.3$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{2051}$ is 43.

Prime numbers less than or equal to 43:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43

Use a calculator to test whether 2051 is divisible by prime numbers less than or equal to 43.

2051 is divisible by 7. Hence, 2051 is a composite number.

(e) 2069 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 2069 is $2+0+6+9=17$ which is not divisible by 3, then 2069 is not divisible by 3.
The last digit of 2069 is neither 0 nor 5, so 2069 is not divisible by 5.

$\sqrt{2069}=45.5$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{2069}$ is 43.

Prime numbers less than or equal to 43:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43

Use a calculator to test whether 2069 is divisible by prime numbers less than or equal to 43.

Since 2069 is not divisible by any of the prime numbers less than or equal to 43, then 2069 is a prime number.

(f) 2163 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 2163 is $2+1+6+3=12$ which is divisible by 3, then 2163 is divisible by 3 (divisibility test for 3).
Hence, 2163 is a composite number.

Solution 3

(a) $8=1\times8$
$\quad=2\times4$

This statement is true, because 2 and 4 are factors of 8. Hence, if 8 is a factor of another number, then 2 and 4 must also be factors of that number.

For example, 8 is a factor of 16. The factors of 16 also include 2 and 4.

$$\begin{aligned}16&=1\times16\\&=2\times8\\&=4\times4\end{aligned}$$

(b) This statement is false.

$$\begin{aligned}18&=1\times18\\&=2\times9\\&=3\times6\end{aligned}$$

$$\begin{aligned}9&=1\times9\\&=3\times3\end{aligned}$$

18 is a multiple of 6. However, 9 is not a multiple of 6.

Solution 4

29 is a prime number, hence its only factors are 1 and 29.

Let $a=1$ and $b=29$.

$a+b=1+29=30$

Solution 5

(a) 2, 3, 5, 7, 11, 13

(b) 2 and 3 $(2+3=5)$
2 and 5 $(2+5=7)$
2 and 11 $(2+11=13)$

Solution 6

2	6174
3	3087
3	1029
7	343
7	49
7	7
	1

$6174=2\times3\times3\times7\times7\times7=2\times3^2\times7^3$

$a=1$, $b=2$, $c=3$

Solution 7

(a)

3	225
3	75
5	25
5	5
	1

$225 = 3\times3\times5\times5 = 3^2\times5^2$

(b) $x = 2$ and $y = 2$

Solution 8

$28 = 2\times2\times7 = 2^2\times7$
$35 = 5\times7$
$45 = 3\times3\times5 = 3^2\times5$

$28\times35\times45 = 2^2\times7\times5\times7\times3^2\times5$
$= 2^2\times3^2\times5^2\times7^2$

$\sqrt{28\times35\times45} = \sqrt{2^2\times3^2\times5^2\times7^2}$
$= 2\times3\times5\times7 = 210$

Solution 9

(a)

2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

$576 = 2\times2\times2\times2\times2\times2\times3\times3 = 2^6\times3^2$

(b) $\sqrt{576} = \sqrt{2^6\times3^2} = 2^3\times3 = 24$

Solution 10

(a)

2	784
2	392
2	196
2	98
7	49
7	7
	1

$784 = 2\times2\times2\times2\times7\times7 = 2^4\times7^2$

(b) $\sqrt{784} = \sqrt{2^4\times7^2} = 2^2\times7 = 28$

Solution 11

(a)

11	34969
11	3179
17	289
17	17
	1

$34969 = 11\times11\times17\times17 = 11^2\times17^2$

(b) $\sqrt{34969\times100} = \sqrt{34969}\times\sqrt{100}$
$= \sqrt{11^2\times17^2}\times\sqrt{10^2} = 11\times17\times10$
$= 2\times5\times11\times17$

Solution 12

(a)

2	4900
2	2450
5	1225
5	245
7	49
7	7
	1

$4900 = 2\times2\times5\times5\times7\times7 = 2^2\times5^2\times7^2$

(b)

2	42
3	21
7	7
	1

$42 = 2\times3\times7$

$$\frac{\sqrt{4900}}{42} = \frac{\sqrt{2^2\times5^2\times7^2}}{2\times3\times7} = \frac{2\times5\times7}{2\times3\times7} = \frac{5}{3} = 1\frac{2}{3}$$

Solution 13

(a)

2	4410
3	2205
3	735
5	245
7	49
7	7
	1

$4410 = 2\times3\times3\times5\times7\times7 = 2\times3^2\times5\times7^2$

(b) $\sqrt{44100} = \sqrt{4410\times10} = \sqrt{2\times3^2\times5\times7^2\times2\times5}$
$= \sqrt{2^2\times3^2\times5^2\times7^2} = 2\times3\times5\times7 = 210$

Solution 14

(a)

2	156
2	78
3	39
13	13
	1

$156 = 2\times2\times3\times13 = 2^2\times3\times13$

(b) $39 = 3\times13$

$\sqrt{156\times39} = \sqrt{2^2\times3\times13\times3\times13}$
$= \sqrt{2^2\times3^2\times13^2} = 2\times3\times13 = 78$

Solution 15

$\sqrt[3]{74088} = \sqrt[3]{(6\times7)^3} = 6\times7 = 42$

Solution 16

(a)

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$9261 = 3\times3\times3\times7\times7\times7 = 3^3\times7^3$

(b) $\sqrt[3]{9261} = \sqrt[3]{3^3\times7^3} = 3\times7 = 21$

Solution 17

(a)

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$5832 = 2\times2\times2\times3\times3\times3\times3\times3\times3$
$= 2^3\times3^6$

(b) $\sqrt[3]{5832} = \sqrt[3]{2^3\times3^6} = 2\times3^2 = 18$

Solution 18

(a)

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$2744 = 2\times2\times2\times7\times7\times7 = 2^3\times7^3$

(b) $\sqrt[3]{2744} = \sqrt[3]{2^3\times7^3} = 2\times7 = 14$

Solution 19

(a)

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$3375 = 3\times3\times3\times5\times5\times5 = 3^3\times5^3$

(b) $\sqrt[3]{-3375} = \sqrt[3]{(-1)(3375)} = \sqrt[3]{(-1)\left(3^3\times5^3\right)}$

$= \sqrt[3]{(-1)\left(3^3\right)\left(5^3\right)} = \left(\sqrt[3]{(-1)}\right)\left(\sqrt[3]{3^3}\right)\left(\sqrt[3]{5^3}\right)$

$= (-1)(3)(5) = -15$

Solution 20

(a)

2	4536
2	2268
2	1134
3	567
3	189
3	63
3	21
7	7
	1

$4536 = 2\times2\times2\times3\times3\times3\times3\times7 = 2^3\times3^4\times7$

$a=3,\ b=4,\ c=1$

(b) First four prime numbers that end with the digit 7:

7, 17, 37, 47

$\text{Sum} = 7+17+37+47 = 108$

Solution 21

(a)

5	1225
5	245
7	49
7	7
	1

$1225 = 5\times5\times7\times7 = 5^2\times7^2$

(b)

(i) $\sqrt{1225} = \sqrt{5^2\times7^2} = 5\times7 = 35$

(ii) $\sqrt[3]{\dfrac{1225\times8}{25\div7}} = \sqrt[3]{\dfrac{5^2\times7^2\times8}{25\div7}} = \sqrt[3]{\dfrac{5^2\times7^2\times2^3}{5^2\div7}}$

$= \sqrt[3]{\dfrac{5^2\times7^2\times2^3\times7}{5^2}} = \sqrt[3]{7^3\times2^3} = 7\times2 = 14$

Solution 22

$\sqrt[3]{-5832000} \div \sqrt{225}$

$= \sqrt[3]{(-1)(5832000)} \div \sqrt{225}$

$= \dfrac{\sqrt[3]{-1}\left(\sqrt[3]{5832000}\right)}{\sqrt{225}} = \dfrac{(-1)\left(\sqrt[3]{2^6\times3^6\times5^3}\right)}{\sqrt{3^2\times5^2}}$

$= \dfrac{(-1)\left(2^2\times3^2\times5\right)}{3\times5} = -\left(2^2\times3\right)$

Solution 23

(a) $\sqrt{51} \approx \sqrt{49} = 7$

(b) $\sqrt{120} \approx \sqrt{121} = 11$

(c) $\sqrt{166} \approx \sqrt{169} = 13$

(d) $\sqrt[3]{28} \approx \sqrt[3]{27} = 3$

(e) $\sqrt[3]{129} \approx \sqrt[3]{125} = 5$

(f) $\sqrt[3]{8050} \approx \sqrt[3]{8000} = 20$

Solution 24

(a)

2	1960
2	980
2	490
5	245
7	49
7	7
	1

$1960 = 2\times2\times2\times5\times7\times7 = 2^3\times5\times7^2$

(b) $1960m = 2^3\times5\times7^2\times m$

For m to be the smallest, $m = 2\times5$ so that
$2^3\times5\times7^2\times m = 2^3\times5\times7^2\times2\times5$
$= 2^4\times5^2\times7^2$ is a perfect square.

Smallest positive integer value of m
$= 2\times5 = 10$

Solution 25

(a)

2	3150
3	1575
3	525
5	175
5	35
7	7
	1

$3150 = 2\times3\times3\times5\times5\times7 = 2\times3^2\times5^2\times7$

(b) $3150n = 2\times3^2\times5^2\times7\times n$

For n to be the smallest, $n = 2\times7$ so that
$2\times3^2\times5^2\times7\times n = 2\times3^2\times5^2\times7\times2\times7$
$= 2^2\times3^2\times5^2\times7^2$ is a perfect square.

Least integer value of $n = 2\times7 = 14$

Solution 26

(a)

2	5040
2	2520
2	1260
2	630
3	315
3	105
5	35
7	7
	1

$5040 = 2\times2\times2\times2\times3\times3\times5\times7 = 2^4\times3^2\times5\times7$

(b) $5040x = 2^4\times3^2\times5\times7\times x$

For x to be the smallest, $x = 5\times7$ so that
$2^4\times3^2\times5\times7\times x = 2^4\times3^2\times5\times7\times5\times7$
$= 2^4\times3^2\times5^2\times7^2$ is a perfect square.

Smallest integer value of $x = 5\times7 = 35$

Solution 27

(a) $\left(2^3\times3^6\times5^2\right)^2 = 2^6\times3^{12}\times5^4$

(b) No, because the index of 5 is not a multiple of 3.

Solution 28

(a)

2	2700
2	1350
3	675
3	225
3	75
5	25
5	5
	1

$2700 = 2\times2\times3\times3\times3\times5\times5 = 2^2\times3^3\times5^2$

(b) Let the number be n.

$2700n = 2^2 \times 3^3 \times 5^2 \times n$

For n to be the smallest, $n = 2 \times 5$ so that
$2^2 \times 3^3 \times 5^2 \times n = 2^2 \times 3^3 \times 5^2 \times 2 \times 5$
$= 2^3 \times 3^3 \times 5^3$ is a perfect cube.

Smallest value of $n = 2 \times 5 = 10$

Solution 29

2	882
3	441
3	147
7	49
7	7
	1

$882 = 2 \times 3 \times 3 \times 7 \times 7 = 2 \times 3^2 \times 7^2$

$882m = 2 \times 3^2 \times 7^2 \times m$

For m to be the smallest, $m = 2^2 \times 3 \times 7$ so that $2 \times 3^2 \times 7^2 \times m = 2 \times 3^2 \times 7^2 \times 2^2 \times 3 \times 7$
$= 2^3 \times 3^3 \times 7^3$ is a perfect cube.

Smallest value of $m = 2^2 \times 3 \times 7 = 84$

Solution 30

2	6534
3	3267
3	1089
3	363
11	121
11	11
	1

$6534 = 2 \times 3 \times 3 \times 3 \times 11 \times 11 = 2 \times 3^3 \times 11^2$

$6534k = 2 \times 3^3 \times 11^2 \times k$

For k to be the smallest, $k = 2^2 \times 11$ so that
$2 \times 3^3 \times 11^2 \times k = 2 \times 3^3 \times 11^2 \times 2^2 \times 11$
$= 2^3 \times 3^3 \times 11^3$ is a perfect cube.

Smallest integer value of $k = 2^2 \times 11 = 44$

Solution 31

(a)

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^6 \times 3^3$

(b) $\sqrt[3]{1728} = \sqrt[3]{2^6 \times 3^3} = 2^2 \times 3 = 12$

(c) $\dfrac{1728}{n} = \dfrac{2^6 \times 3^3}{n}$

For n to be the smallest, $n = 3$ so that
$\dfrac{2^6 \times 3^3}{n} = \dfrac{2^6 \times 3^3}{3} = 2^6 \times 3^2$ is a perfect square.

Smallest value of $n = 3$

Solution 32

(a)

2	336
2	168
2	84
2	42
3	21
7	7
	1

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7$

(b)

2	180
2	90
3	45
3	15
5	5
	1

$180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$

$336 = 2^4 \times 3 \times 7$
$336 = 2^2 \times 2^2 \times 3 \times 7$
$336 \times 3 = 2^2 \times 2^2 \times 3 \times 3 \times 7$
$336 \times 3 \times 5 = 2^2 \times 2^2 \times 3^2 \times 5 \times 7$
$336 \times 3 \times 5 = 180 \times 2^2 \times 7$

Smallest integer value of $p = 3 \times 5 = 15$

Solution 33

(a)

2	60
2	30
3	15
5	5
	1

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

3	825
5	275
5	55
11	11
	1

$825 = 3 \times 5 \times 5 \times 11 = 3 \times 5^2 \times 11$

(b) $60 = 2^2 \times 3 \times 5$
$60 \times 5 = 2^2 \times 3 \times 5^2$
$60 \times 5 \times 11 = 2^2 \times 3 \times 5^2 \times 11$
$60 \times 5 \times 11 = 2^2 \times 825$

Smallest positive integer value of n
$= 5 \times 11 = 55$

Solution 34

(a)

3	1521
3	507
13	169
13	13
	1

$1521 = 3 \times 3 \times 13 \times 13 = 3^2 \times 13^2$

(b) $\sqrt{1521} = \sqrt{3^2 \times 13^2} = 3 \times 13 = 39$

(c) $1521 = 3 \times 3 \times 13 \times 13$
$1521 \times 2 = 2 \times 3 \times 3 \times 13 \times 13$
$1521 \times 2 = 6 \times 3 \times 13 \times 13$

Smallest integer value of $k = 2$

Solution 35

(a)

2	108
2	54
3	27
3	9
3	3
	1

$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$

(b) $108 = 2^2 \times 3^3$
$108 \times 2 = 2^3 \times 3^3$
$108 \times 2 = 2^3 \times 3^2 \times 3$
$108 \times 2 \times 5 = 2^3 \times 3^2 \times 3 \times 5$
$108 \times 2 \times 5 = 360 \times 3$

Smallest positive value of $x = 2 \times 5 = 10$

(c) $360y = 2^3 \times 3^2 \times 5 \times y$

For y to be the smallest, $y = 3 \times 5^2$ so that
$2^3 \times 3^2 \times 5 \times y = 2^3 \times 3^2 \times 5 \times 3 \times 5^2$
$= 2^3 \times 3^3 \times 5^3$ is a perfect cube.

Smallest positive integer value of y
$= 3 \times 5^2 = 75$

Solution 36

(a) $63 = 1 \times 63$
$= 3 \times 21$
$= 7 \times 9$

Second largest factor $= 21$

(b) LCM of 3 and $4 = 12$

100^{th} common multiple $= 12 \times 100 = 1200$

Solution 37

Prime numbers between 1 and 14:

2, 3, 5, 7, 11, 13

LCM $= 2 \times 3 \times 5 \times 7 \times 11 \times 13 = 30030$

Solution 38

(a)

2	84
2	42
3	21
7	7
	1

$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$

2	126
3	63
3	21
7	7
	1

$126 = 2 \times 3 \times 3 \times 7 = 2 \times 3^2 \times 7$

(b)
(i) HCF of 84 and $126 = 2 \times 3 \times 7 = 42$

(ii) LCM of 84 and $126 = 2^2 \times 3^2 \times 7 = 252$

Solution 39

2	102
3	51
17	17
	1

$102 = 2 \times 3 \times 17$

2	138
3	69
23	23
	1

$138 = 2 \times 3 \times 23$

HCF of 102 and $138 = 2 \times 3 = 6$

LCM of 102 and $138 = 2 \times 3 \times 17 \times 23 = 2346$

Solution 40

2	112
2	56
2	28
2	14
7	7
	1

$112 = 2 \times 2 \times 2 \times 2 \times 7 = 2^4 \times 7$

2	280
2	140
2	70
5	35
7	7
	1

$280 = 2 \times 2 \times 2 \times 5 \times 7 = 2^3 \times 5 \times 7$

HCF of 112 and $280 = 2^3 \times 7 = 56$

LCM of 112 and $280 = 2^4 \times 5 \times 7 = 560$

Solution 41

$14 = 2 \times 7$
$27 = 3 \times 3 \times 3 = 3^3$
$63 = 3 \times 3 \times 7 = 3^2 \times 7$

There is no HCF.

LCM of 14, 27 and $63 = 2 \times 3^3 \times 7 = 378$

Solution 42

$HCF = 3^3 \times 13$

$LCM = 2^4 \times 3^5 \times 5^2 \times 7^2 \times 13^2$

Solution 43

2	1260
2	630
3	315
3	105
5	35
7	7
	1

$1260 = 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2^2 \times 3^2 \times 5 \times 7$

$HCF = 2^2 \times 3 \times 7$

$LCM = 2^3 \times 3^2 \times 5 \times 7^2 \times 11^2$

Solution 44

2	54
3	27
3	9
3	3
	1

$54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$

2	126
3	63
3	21
7	7
	1

$126 = 2 \times 3 \times 3 \times 7 = 2 \times 3^2 \times 7$

HCF: $18 = 2 \times 3^2$
LCM: $2^2 \times 3^3 \times 7 \times 11$

For the third number:

2^2 is a factor, because it is not a factor of 54 and 126 but is included in the LCM.

3^2 is a factor, because it is included in the HCF, which means that all three numbers must have it as a factor.

11 is a factor, because it is not a factor of 54 and 126 but is included in the LCM.

Third number $= 2^2 \times 3^2 \times 11 = 396$

Solution 45

HCF of 168 and $324 = 2^2 \times 3 = 12$

Solution 46

$HCF = 2^3 \times 3^2 = 72$

$LCM = 2^4 \times 3^3 \times 5 \times 7 = 15120$

Solution 47

$HCF = 2^2 \times 3 \times 13 \times 19^2$

$LCM = 2^3 \times 3^3 \times 5^2 \times 7^3 \times 13^2 \times 17 \times 19^3$

Solution 48

(a)

2	90
3	45
3	15
5	5
	1

$90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$

2	126
3	63
3	21
7	7
	1

$126 = 2 \times 3 \times 3 \times 7 = 2 \times 3^2 \times 7$

2	198
3	99
3	33
11	11
	1

$198 = 2 \times 3 \times 3 \times 11 = 2 \times 3^2 \times 11$

HCF of 90, 126 and $198 = 2 \times 3^2 = 18$

(b) $2^3 \times 3^2 \times 5 \times 7$
$2^2 \times 3^3 \times 5^2 \times 11$
$2^2 \times 3 \times 5 \times 11$

$\text{LCM} = 2^3 \times 3^3 \times 5^2 \times 7 \times 11$

Solution 49

(a) HCF of 1575 and $42 = 3 \times 7 = 21$

(b) $42 = 2 \times 3 \times 7$
$42 \times 3 = 2 \times 3^2 \times 7$
$42 \times 3 \times 5^2 = 2 \times 3^2 \times 5^2 \times 7$
$42 \times 3 \times 5^2 = 2 \times 1575$

Smallest positive integer value of n
$= 3 \times 5^2 = 75$

Solution 50

(a)

2	126
3	63
3	21
7	7
	1

$126 = 2 \times 3 \times 3 \times 7 = 2 \times 3^2 \times 7$

(b) $35 = 5 \times 7$

$126 = 2 \times 3^2 \times 7$
$126 \times 5 = 2 \times 3^2 \times 7 \times 5$
$126 \times 5 = 2 \times 3^2 \times 35$

Smallest positive integer value of $n = 5$

(c) $6 = 2 \times 3$
$14 = 2 \times 7$
$126 = 2 \times 3^2 \times 7$

Possible values of x :

$x = 3^2 = 9$
$x = 3^2 \times 7 = 63$

Solution 51

(a) HCF of A and $B = 2^2 \times 3$

(b) LCM of A and $B = 2^2 \times 3^3 \times 5^2 \times 7^4$

(c) $A \times B = 2^2 \times 3 \times 7^4 \times 2^2 \times 3^3 \times 5^2$
$= 2^4 \times 3^4 \times 5^2 \times 7^4$

$\sqrt{A \times B} = \sqrt{2^4 \times 3^4 \times 5^2 \times 7^4} = 2^2 \times 3^2 \times 5 \times 7^2$

Solution 52

(a) HCF of A and $B = 2^2 \times 3^4 \times 11^2$

(b) LCM of A and $C = 2^2 \times 3^4 \times 5^4 \times 7^2 \times 11^3$

Solution 53

(a)
(i)

2	378
3	189
3	63
3	21
7	7
	1

$378 = 2\times3\times3\times3\times7 = 2\times3^3\times7$

(ii)

2	1008
2	504
2	252
2	126
3	63
3	21
7	7
	1

$1008 = 2\times2\times2\times2\times3\times3\times7 = 2^4\times3^2\times7$

(b) HCF of 378 and $1008 = 2\times3^2\times7 = 126$

Solution 54

(a) $\text{HCF} = x\times y\times z = xyz$

(b) $\text{LCM} = w^2\times x^3\times y^2\times z = w^2x^3y^2z$

(c) $\text{HCF}+\text{LCM} = 24060$

$xyz + w^2x^3y^2z = 24060$

$(5)(4)z + \left(2^2\right)\left(5^3\right)\left(4^2\right)z = 24060$

$20z + 8000z = 24060$

$8020z = 24060$

$z = \dfrac{24060}{8020} = 3$

Solution 55

(a)

3	63
3	21
7	7
	1

$63 = 3\times3\times7 = 3^2\times7$

3	105
5	35
7	7
	1

$105 = 3\times5\times7$

2	420
2	210
3	105
5	35
7	7
	1

$420 = 2\times2\times3\times5\times7 = 2^2\times3\times5\times7$

HCF of 63, 105 and $420 = 3\times7$

LCM of 63, 105 and $420 = 2^2\times3^2\times5\times7$

(b) $\text{HCF} = 3\times7\times a\times c = 21ac$

$\text{LCM} = 2^2\times3^2\times5\times7\times a^5\times b^3\times c^3$
$= 1260a^5b^3c^3$

Solution 56

3	135
3	45
3	15
5	5
	1

$135 = 3\times3\times3\times5 = 3^3\times5$

2	120
2	60
2	30
3	15
5	5
	1

$120 = 2\times2\times2\times3\times5 = 2^3\times3\times5$

2	190
5	95
19	19
	1

$190 = 2\times5\times19$

HCF of 135, 120 and 190 $= 5$

LCM of 135, 120 and 190 $= 2^3\times3^3\times5\times19$

$$\frac{\text{LCM}}{\text{HCF}} = \frac{2^3\times3^3\times5\times19}{5} = 2^3\times3^3\times19 = 4104$$

Solution 57

(a)

2	90
3	45
3	15
5	5
	1

$90 = 2\times3\times3\times5 = 2\times3^2\times5$

(b) $6 = 2\times3$
$15 = 3\times5$

$90 = 2\times3^2\times5$

Since x is an odd number, it will not have 2 as a factor.

Possible values of x:
$x = 3^2 = 9$
$x = 3^2\times5 = 45$

Solution 58

(a)

2	504
2	252
2	126
3	63
3	21
7	7
	1

$504 = 2\times2\times2\times3\times3\times7 = 2^3\times3^2\times7$

2	972
2	486
3	243
3	81
3	27
3	9
3	3
	1

$972 = 2\times2\times3\times3\times3\times3\times3 = 2^2\times3^5$

(b) HCF of 504 and 972 $= 2^2\times3^2 = 36$

(c) $\sqrt[3]{972m} = \sqrt[3]{2^2\times3^5\times m}$

For m to be the smallest, $m = 2\times3$ so that
$2^2\times3^5\times m = 2^2\times3^5\times2\times3$
$= 2^3\times3^6$ is a perfect cube.

Smallest positive integer value of m
$= 2\times3 = 6$

Solution 59

(a)

2	1500
2	750
3	375
5	125
5	25
5	5
	1

$1500 = 2\times2\times3\times5\times5\times5 = 2^2\times3\times5^3$

$1500x = 2^2 \times 3 \times 5^3 \times x$

For x to be the smallest, $x = 3 \times 5$ so that
$2^2 \times 3 \times 5^3 \times x = 2^2 \times 3 \times 5^3 \times 3 \times 5$
$= 2^2 \times 3^2 \times 5^4$ is a perfect square.

Smallest possible value of $x = 3 \times 5 = 15$

(b) $\text{LCM} = 2^2 \times 3^2 \times 5^3 \times 7$

Solution 60

(a) HCF of 180 and $784 = 2^2 = 4$

(b) $\sqrt[3]{180 \times 784 \times m} = \sqrt[3]{2^2 \times 3^2 \times 5 \times 2^4 \times 7^2 \times m}$
$= \sqrt[3]{2^6 \times 3^2 \times 5 \times 7^2 \times m}$

For m to be the smallest, $m = 3 \times 5^2 \times 7$ so that
$2^6 \times 3^2 \times 5 \times 7^2 \times m = 2^6 \times 3^2 \times 5 \times 7^2 \times 3 \times 5^2 \times 7$
$= 2^6 \times 3^3 \times 5^3 \times 7^3$ is a perfect cube.

Smallest positive integer value of m
$= 3 \times 5^2 \times 7 = 525$

(c) $180 = 2^2 \times 3^2 \times 5$
$180 \times 2^2 = 2^4 \times 3^2 \times 5$
$180 \times 2^2 = 16 \times 3^2 \times 5$

Smallest positive integer value of $n = 2^2 = 4$

Solution 61

(a)

2	1400
2	700
2	350
5	175
5	35
7	7
	1

$1400 = 2 \times 2 \times 2 \times 5 \times 5 \times 7 = 2^3 \times 5^2 \times 7$

(b) $1400m = 2^3 \times 5^2 \times 7 \times m$

For m to be the smallest, $m = 2 \times 7$ so that
$2^3 \times 5^2 \times 7 \times m = 2^3 \times 5^2 \times 7 \times 2 \times 7$
$= 2^4 \times 5^2 \times 7^2$ is a perfect square.

Smallest positive integer value of m
$= 2 \times 7 = 14$

(c) HCF of 560 and $1400 = 2^3 \times 5 \times 7 = 280$

Solution 62

(a)

2	180
2	90
3	45
3	15
5	5
	1

$180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$

(b) $180k = 2^2 \times 3^2 \times 5 \times k$

For k to be the smallest, $k = 2 \times 3 \times 5^2$ so that
$2^2 \times 3^2 \times 5 \times k = 2^2 \times 3^2 \times 5 \times 2 \times 3 \times 5^2$
$= 2^3 \times 3^3 \times 5^3$ is a perfect cube.

Smallest value of $k = 2 \times 3 \times 5^2 = 150$

(c) HCF of 180 and $576 = 2^2 \times 3^2 = 36$

Solution 63

(a) LCM of 162 and $98 = 2 \times 3^4 \times 7^2$

(b) $162 \times 98 = 15876$

$\sqrt{15876} = \sqrt{162 \times 98} = \sqrt{2 \times 3^4 \times 2 \times 7^2}$
$= \sqrt{2^2 \times 3^4 \times 7^2} = 2 \times 3^2 \times 7$

Solution 64

(a) HCF of 504 and 972 $= 2^2 \times 3^2 = 36$

(b) $504 = 2^3 \times 3^2 \times 7$
$504 = 2 \times 2^2 \times 3^2 \times 7$
$504 \times 3^3 = 2 \times 2^2 \times 3^5 \times 7$
$504 \times 3^3 = 2 \times 7 \times 972$

Smallest positive integer value of n
$= 3^3 = 27$

Solution 65

(a) HCF of 168 and 378 $= 2 \times 3 \times 7 = 42$

(b) $168 = 2^3 \times 3 \times 7$
$168 = 2^2 \times 2 \times 3 \times 7$
$168 \times 3^2 = 2^2 \times 2 \times 3^3 \times 7$
$168 \times 3^2 = 2^2 \times 378$

Smallest positive integer value of $n = 3^2 = 9$

(c) $\sqrt{168 \times 378} = \sqrt{2^3 \times 3 \times 7 \times 2 \times 3^3 \times 7}$
$= \sqrt{2^4 \times 3^4 \times 7^2} = 2^2 \times 3^2 \times 7 = 252$

Problems in Real-World Contexts

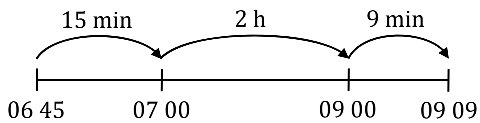
Solution 66

$4 = 2^2$
$16 = 2^4$
$18 = 2 \times 3^2$

LCM of 4, 16 and 18 $= 2^4 \times 3^2 = 144$

The three bells will chime together again after 144 minutes.

$144 \text{ min} = 2 \text{ h } 24 \text{ min}$



All three bells will chime together again at 09 09.

Solution 67

(a) $12 = 2 \times 2 \times 3 = 2^2 \times 3$
$30 = 2 \times 3 \times 5$
$66 = 2 \times 3 \times 11$

LCM of 12, 30 and 66 $= 2^2 \times 3 \times 5 \times 11 = 660$

The three bells will next toll together again after 660 seconds.

$$660 \text{ s} = \frac{660}{60} \text{ min} = 11 \text{ min}$$

The three bells will next toll together again at 6.11 a.m.

(b) The three bells will toll together at 6 a.m., 6.11 a.m., 6.22 a.m., 6.33 a.m., 6.44 a.m., 6.55 a.m.

Hence, there is a total of 6 times.

Solution 68

3	45
3	15
5	5
	1

$45 = 3 \times 3 \times 5 = 3^2 \times 5$

2	60
2	30
3	15
5	5
	1

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

2	120
2	60
2	30
3	15
5	5
	1

$120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5$

LCM of 45, 60 and 120 $= 2^3 \times 3^2 \times 5 = 360$

The lights will next turn green together after 360 seconds.

$$360 \text{ s} = \frac{360}{60} \text{ min} = 6 \text{ min}$$

The lights will next turn green together at 10 06.

Solution 69

$1 \text{ min} = 60 \text{ s}$
$1 \text{ min } 10 \text{ s} = 70 \text{ s}$
$1 \text{ min } 18 \text{ s} = 78 \text{ s}$

2	60
2	30
3	15
5	5
	1

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

2	70
5	35
7	7
	1

$70 = 2 \times 5 \times 7$

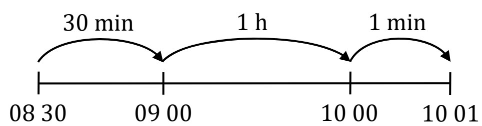
2	78
3	39
13	13
	1

$78 = 2 \times 3 \times 13$

LCM of 60, 70 and 78 $= 2^2 \times 3 \times 5 \times 7 \times 13$
$= 5460$

All the traffic lights will turn red at the same time again after 5460 seconds.

$$5460\text{ s} = \frac{5460}{60}\text{ min} = 91\text{ min} = 1\text{ h } 31\text{ min}$$



All the traffic lights will turn red at the same time again at 10 01.

Solution 70

(a) $20 = 2 \times 2 \times 5 = 2^2 \times 5$
$30 = 2 \times 3 \times 5$
$45 = 3 \times 3 \times 5 = 3^2 \times 5$

LCM of 20, 30 and 45 $= 2^2 \times 3^2 \times 5 = 180$

They will next flash together after 180 seconds.

$$180\text{ s} = \frac{180}{60}\text{ min} = 3\text{ min}$$

They will next flash together at 7.03 p.m.

(b) $6\text{ h} = (6 \times 60)\text{ min} = 360\text{ min}$

$$\text{Number of times} = \frac{360}{3} = 120$$

Including the first, they will flash 121 times together.

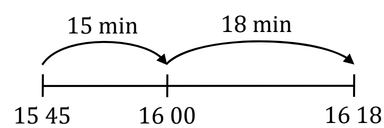
Solution 71

$20 = 2 \times 2 \times 5 = 2^2 \times 5$
$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$
$55 = 5 \times 11$

LCM of 20, 36 and 55 $= 2^2 \times 3^2 \times 5 \times 11 = 1980$

All three neon lights will flash together again after 1980 seconds.

$$1980\text{ s} = \frac{1980}{60}\text{ min} = 33\text{ min}$$



All three neon lights will flash together again at 16 18.

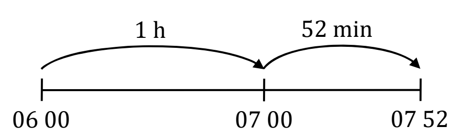
Solution 72

$8 = 2^3$
$14 = 2 \times 7$
$16 = 2^4$

LCM of 8, 14 and 16 $= 2^4 \times 7 = 112$

The three services will next leave the depot together after 112 minutes.

$112\text{ min} = 1\text{ h } 52\text{ min}$



The three services will next leave the depot together at 07 52.

Solution 73

(a) $12 = 2^2 \times 3$

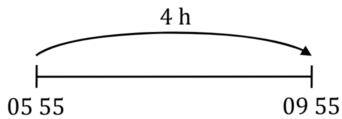
$16 = 2^4$

$20 = 2^2 \times 5$

LCM of 12, 16 and 20 $= 2^4 \times 3 \times 5 = 240$

(b) The buses on each of the three routes will leave together again after 240 minutes.

$240 \text{ min} = 4 \text{ h}$



The buses on each of the three routes will leave together again at 09 55.

Solution 74

$35 = 5 \times 7$

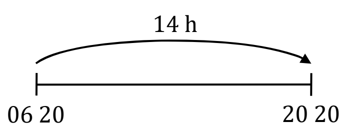
$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$

$42 = 2 \times 3 \times 7$

LCM of 35, 40 and 42 $= 2^3 \times 3 \times 5 \times 7 = 840$

The 3 buses will next leave the interchange together after 840 minutes.

$840 \text{ min} = \dfrac{840}{60} \text{ h} = 14 \text{ h}$



The 3 buses will next leave the interchange together at 20 20.

Solution 75

$6 = 2 \times 3$

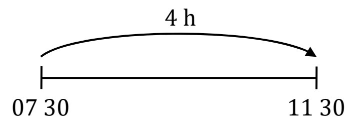
$16 = 2^4$

$20 = 2^2 \times 5$

LCM of 3, 6, 16 and 20 $= 2^4 \times 3 \times 5 = 240$

The four services will next leave the depot together again after 240 minutes.

$240 \text{ min} = 4 \text{ h}$



The four services will next leave the depot together again at 11 30.

Solution 76

$1 \text{ h} = 60 \text{ min}$

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

$30 = 2 \times 3 \times 5$

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

LCM of 12, 30 and 60 $= 2^2 \times 3 \times 5 = 60$

The buses for the three services will next leave the hotel at the same time after 60 minutes, i.e. 1 hour, at 10 30.

Solution 77

(a) $14 = 2 \times 7$

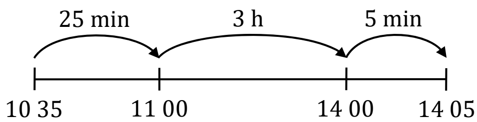
$21 = 3 \times 7$

$35 = 5 \times 7$

LCM of 14, 21 and 35 $= 2 \times 3 \times 5 \times 7 = 210$

210 minutes would have passed when the three services next leave the hospital at the same time again.

(b) $210\text{ min} = 3\text{ h } 30\text{ min}$



The time shown on his watch was 14 05.

Solution 78

(a) $8 = 2^3$

$14 = 2 \times 7$

$28 = 2 \times 2 \times 7 = 2^2 \times 7$

LCM of 8, 14 and $28 = 2^3 \times 7 = 56$

(b) $4.5\text{ hours} = 4.5 \times 60 = 270\text{ min}$

$$270 \div 56 = 4\frac{23}{28}$$

Not counting the first, the 3 sprinklers will go off 4 times simultaneously if they are left on for 4.5 hours.

Solution 79

(a)

2	72
2	36
2	18
3	9
3	3
	1

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

2	90
3	45
3	15
5	5
	1

$90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$

LCM of 72 and $90 = 2^3 \times 3^2 \times 5 = 360$

They will meet at the starting point again after 360 s.

(b) Number of laps Alvin will run $= \frac{360}{72} = 5$

Number of laps Henry will run $= \frac{360}{90} = 4$

Solution 80

$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

$78 = 2 \times 3 \times 13$

LCM of 48, 60 and $78 = 2^4 \times 3 \times 5 \times 13 = 3120$

$$3120\text{ s} = \frac{3120}{60}\text{ min} = 52\text{ min}$$

Solution 81

(a)

2	168
2	84
2	42
3	21
7	7
	1

$168 = 2 \times 2 \times 2 \times 3 \times 7 = 2^3 \times 3 \times 7$

2	132
2	66
3	33
11	11
	1

$132 = 2 \times 2 \times 3 \times 11 = 2^2 \times 3 \times 11$

2	84
2	42
3	21
7	7
	1

$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$

HCF of 168, 132 and $84 = 2^2 \times 3 = 12$

Longest possible length of each side of a cube
$=12\,\text{cm}$

Volume of the cube $=12^3=1728\,\text{cm}^3$

(b) Number of cubes $=\left(\dfrac{168}{12}\right)\left(\dfrac{132}{12}\right)\left(\dfrac{84}{12}\right)$
$=(14)(11)(7)=1078$

Solution 82

2	112
2	56
2	28
2	14
7	7
	1

$112=2\times2\times2\times2\times7=2^4\times7$

2	98
7	49
7	7
	1

$98=2\times7\times7=2\times7^2$

2	84
2	42
3	21
7	7
	1

$84=2\times2\times3\times7=2^2\times3\times7$

HCF of 112, 98 and 84 $=2\times7=14$

Largest possible value of $L=14$

Solution 83

2	126
3	63
3	21
7	7
	1

$126=2\times3\times3\times7=2\times3^2\times7$

2	140
2	70
5	35
7	7
	1

$140=2\times2\times5\times7=2^2\times5\times7$

2	154
7	77
11	11
	1

$154=2\times7\times11$

HCF of 126, 140 and 154 $=2\times7=14$

Greatest possible length $=14\,\text{cm}$

Solution 84

$45=3\times3\times5=3^2\times5$
$21=3\times7$
$15=3\times5$

LCM of 45, 21 and 15 $=3^2\times5\times7=315$

Smallest possible value of $d=315$

Solution 85

(a)

2	120
2	60
2	30
3	15
5	5
	1

$120=2\times2\times2\times3\times5=2^3\times3\times5$

2	96
2	48
2	24
2	12
2	6
3	3
	1

$96 = 2\times2\times2\times2\times2\times3 = 2^5\times3$

LCM of 120 and $96 = 2^5\times3\times5 = 480$

Shortest possible distance between the first and the last pot of plant
$= 480\text{ m}$

(b) Total number of lamp posts and pots of bougainvillea plants
$= \left(\frac{480}{120}+1\right)+\left(\frac{480}{96}+1\right) = 5+6 = 11$

Solution 86

(a) $15 = 3\times5$
$20 = 2^2\times5$
$30 = 2\times3\times5$

LCM of 15, 20 and $30 = 2^2\times3\times5 = 60$

The three objects will next appear in line 60 m from the beginning of the straight road.

(b) $0.5\text{ km} = 500\text{ m}$

$\frac{500}{60} = 8\frac{1}{3}$

The objects will appear in line 9 times, including the first time.

Solution 87

(a)

2	56
2	28
2	14
7	7
	1

$56 = 2\times2\times2\times7 = 2^3\times7$

2	70
5	35
7	7
	1

$70 = 2\times5\times7$

2	84
2	42
3	21
7	7
	1

$84 = 2\times2\times3\times7 = 2^2\times3\times7$

LCM of 56, 70 and $84 = 2^3\times3\times5\times7 = 840$

The next 3 red objects will be spotted again at 840 m from the initial position.

(b) Number of red lamp posts $= \frac{5000}{840} = 5\frac{20}{21}$

Including the first lamp post, 6 of them are painted red.

(c) Lamp posts: $\frac{5000}{70} = 71\frac{3}{7}$

Dustbins: $\frac{5000}{56} = 89\frac{2}{7}$

Benches: $\frac{5000}{84} = 59\frac{11}{21}$

Including the first, there are 72 lamp posts, 90 dustbins and 60 benches.

Solution 88

(a) $14 = 2\times7$
$21 = 3\times7$
$28 = 2\times2\times7 = 2^2\times7$

LCM of 14, 21 and $28 = 2^2\times3\times7 = 84$

The inter-class games for the 3 levels will be next held simultaneously after 84 days.

(b) Number of days in a year $= 365$

$\frac{365}{84} = 4\frac{29}{84}$

Maximum number of times $= 5$

Solution 89

Number of orange-flavoured lollipops left
$= 109 - 1 = 108$

Number of cola-flavoured lollipops left
$= 85 - 1 = 84$

2	120
2	60
2	30
3	15
5	5
	1

$120 = 2\times2\times2\times3\times5 = 2^3\times3\times5$

2	108
2	54
3	27
3	9
3	3
	1

$108 = 2\times2\times3\times3\times3 = 2^2\times3^3$

2	84
2	42
3	21
7	7
	1

$84 = 2\times2\times3\times7 = 2^2\times3\times7$

HCF of 120, 108 and $84 = 2^2\times3 = 12$

Largest number of packets $= 12$

Number of cola-flavoured lollipops in each packet
$= \frac{84}{12} = 7$

Solution 90

$18 = 2\times3\times3 = 2\times3^2$
$36 = 2\times2\times3\times3 = 2^2\times3^2$
$54 = 2\times3\times3\times3 = 2\times3^3$

HCF of 18, 36 and $54 = 2\times3^2 = 18$

Solution 91

2	220
2	110
5	55
11	11
	1

$220 = 2\times2\times5\times11 = 2^2\times5\times11$

2	300
2	150
3	75
5	25
5	5
	1

$300 = 2\times2\times3\times5\times5 = 2^2\times3\times5^2$

HCF of 220 and $300 = 2^2\times5 = 20$

Solution 92

(a)

2	98
7	49
7	7
	1

$98 = 2 \times 7 \times 7 = 2 \times 7^2$

2	42
3	21
7	7
	1

$42 = 2 \times 3 \times 7$

2	140
2	70
5	35
7	7
	1

$140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

HCF of 98, 42 and $140 = 2 \times 7 = 14$

(b) Total number of sweets each student received

$= \frac{98}{14} + \frac{42}{14} + \frac{140}{14} = 7 + 3 + 10 = 20$

Solution 93

Number of girls left $= 33 - 1 = 32$

Number of boys left $= 58 - 2 = 56$

$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$56 = 2 \times 2 \times 2 \times 7 = 2^3 \times 7$

HCF of 32 and $56 = 2^3 = 8$

Largest number of groups that could be formed

$= 8$

Solution 94

(a)

2	60
2	30
3	15
5	5
	1

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

3	105
5	35
7	7
	1

$105 = 3 \times 5 \times 7$

HCF of 60 and $105 = 3 \times 5 = 15$

(b) Number of girls in each group $= \frac{60}{15} = 4$

Number of boys in each group $= \frac{105}{15} = 7$

Solution 95

(a) $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$18 = 2 \times 3 \times 3 = 2 \times 3^2$

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

HCF of 24, 18 and $12 = 2 \times 3 = 6$

Greatest number of groups that could be formed

$= 6$

(b) Number of Korean students in each group

$= \frac{12}{6} = 2$

Solution 96

Number of girl guides left $= 145 - 1 = 144$

2	144
2	72
2	36
2	18
3	9
3	3
	1

$144 = 2\times2\times2\times2\times3\times3 = 2^4\times3^2$

2	306
3	153
3	51
17	17
	1

$306 = 2\times3\times3\times17 = 2\times3^2\times17$

HCF of 144 and $306 = 2\times3^2 = 18$

Largest number of groups that could be formed
$= 18$

(b) $x = \dfrac{144}{18} = 8$

$y = \dfrac{306}{18} = 17$

Solution 97

2	168
2	84
2	42
3	21
7	7
	1

$168 = 2\times2\times2\times3\times7 = 2^3\times3\times7$

$\text{HCF} = 3$

$\text{LCM} = 2^3\times3\times7$

Since the HCF is 3, their ages must be multiples of 3.

Possible ages:

$2^3\times3\times7 = 168$
$2^3\times3 = 24$
$2^2\times3\times7 = 84$
$2\times3\times7 = 42$
$3\times7 = 21$

By observation, only 21 and 24 have a difference of 3.

Samuel's age $= 24$

Chris's age $= 21$

Solution 98

Time taken by the security guard in Block A for each cycle of patrol
$= 30 + 18 = 48$ min

$48 = 2\times2\times2\times2\times3 = 2^4\times3$
$40 = 2\times2\times2\times5 = 2^3\times5$

LCM of 48 and $40 = 2^4\times3\times5 = 240$

They will next start their patrol at the same time again after 240 minutes.

$240 \text{ min} = \dfrac{240}{60} \text{ h} = 4 \text{ h}$

They will next start their patrol at the same time again at 11 p.m.

Solution 99

LCM of 3, 5 and $11 = 3\times5\times11 = 165$

Smallest possible number of egg tarts baked
$= 165 - 1 = 164$

Solution 100

$18 = 2 \times 3^2$
$25 = 5^2$
$30 = 2 \times 3 \times 5$

LCM of 18, 25 and $30 = 2 \times 3^2 \times 5^2 = 450$

Smallest possible number of magazines in the store
$= 450 + 7 = 457$

Solution 101

(a) $300 = 2^2 \times 3 \times 5^2$
$200 = 2^3 \times 5^2$
$160 = 2^5 \times 5$

HCF of 300, 200 and $160 = 2^2 \times 5 = 20$

Greatest number of prizes $= 20$

(b) Number of vouchers in a prize
$= \frac{300}{20} + \frac{200}{20} + \frac{160}{20} = 15 + 10 + 8 = 33$

(c) Total value of the vouchers in a prize
$= (15 \times \$2) + (10 \times \$5) + (8 \times \$10)$
$= \$30 + \$50 + \$80 = \$160$
